Tips for Teachers

Chapter 2 - Fractional Notation: Multiplication and Division

Section 2.5 - Simplifying

Multiplying by 1

Students will be able to simplify fractional notation more readily if they are first introduced to the idea of multiplying a fraction by 1. Explain to them that any nonzero number divided by itself is 1. When we multiply a number by 1, we get an equivalent form of that number with a different name. For example,

$$\frac{2}{3} = \frac{2}{3} \cdot \frac{1}{1} = \frac{2}{3} \cdot \frac{5}{5} = \frac{10}{15}.$$

$\frac{2}{3}$ and $\frac{10}{15}$ name the same number, and they are equivalent.

Removing a Factor of 1

To simplify a fraction, we reverse the process of multiplying by 1 and *remove* a factor of 1. To do this, we first factor the numerator and the denominator of the fraction, identify the factors that are common to them, and proceed as shown below. Encourage your students to write each of these steps as they do their homework, since skipping steps can lead to mistakes.

$$\begin{aligned}
\frac{24}{20} &= \frac{6 \cdot 4}{5 \cdot 4} && \text{Factoring the numerator and the denominator} \\
&= \frac{6}{5} \cdot \frac{4}{4} && \text{Factoring the fraction} \\
&= \frac{6}{5} \cdot 1 && \frac{4}{4} = 1 \\
&= \frac{6}{5} && \text{Removing a factor of 1}
\end{aligned}$$

Canceling

The idea of canceling can be introduced as a means of streamlining the process of removing a factor of 1. Emphasize to your students that, by keeping in mind the idea of removing a factor of 1, they will be led to cancel correctly. That is, canceling can be done only when the numerator and denominator have common factors. After doing an exercise like the one above in class, show how it could also be done using canceling.

$$\begin{aligned}
\frac{24}{20} &= \frac{6 \cdot 4}{5 \cdot 4} && \text{Factoring the numerator and the denominator} \\
&= \frac{6 \cdot \cancel{4}}{5 \cdot \cancel{4}} && \text{Removing a factor of 1 by canceling the factor of 4 that is common to the numerator and the denominator} \\
&= \frac{6}{5}
\end{aligned}$$

Supplement Key
Further Instruction and Practice for Your Students

Video	Audio cassette	InterAct Math Online Exercises	Printed Test Bank/Instructor's Resource Guide	
			Exercises	Chapter Review
Tape 4	4A		p. 621	p. 688

Sections 2.6 and 2.7 - Multiplying, Dividing, and Simplifying

Students often find operations with fractional notation to be intimidating. By outlining and demonstrating steps to follow when doing these operations, an instructor can go far toward allaying a student's apprehensions. Here we consider the operations of multiplication and division.

Multiplying and Dividing with Fractional Notation

To multiply and simplify:
a) Multiply the numerators and multiply the denominators, but do not carry out the products.
b) Factor the numerator and the denominator.
c) Factor the fraction to remove factors of 1. (See Tips for Teachers for Section 2.5.)
d) Carry out the remaining products.

To divide and simplify, multiply the dividend by the reciprocal of the divisor. Then proceed as described above to complete the process of multiplying.

In class do as many examples as possible, describing each step as it is done. We illustrate here with a division problem.

Divide and simplify: $\frac{16}{21} \div \frac{10}{7}$.

$$\begin{aligned}
\frac{16}{21} \div \frac{10}{7} &= \frac{16}{21} \cdot \frac{7}{10} && \text{Multiplying by the reciprocal of the divisor} \\
&= \frac{16 \cdot 7}{21 \cdot 10} && \text{Multiplying the numerators and the denominators} \\
&= \frac{2 \cdot 8 \cdot 7}{3 \cdot 7 \cdot 2 \cdot 5} && \text{Factoring the numerator and the denominator} \\
&= \frac{8}{3 \cdot 5} \cdot \frac{2 \cdot 7}{2 \cdot 7} && \text{Factoring the fraction to remove a factor of 1} \\
&= \frac{8}{3 \cdot 5} \cdot 1 \\
&= \frac{8}{15} && \text{Carrying out the remaining product}
\end{aligned}$$

Students should be encouraged to recite each step as they multiply or divide and simplify. They should also be advised not to skip steps.

Canceling

Many students will want to remove a factor of 1 by canceling. Caution them that they can cancel only when the numerator and the denominator have common factors. (See Tips for Teachers for Section 2.5.) Do at least one of your examples again using canceling to reinforce this.

Supplement Key
Further Instruction and Practice for Your Students

Video	Audio cassette	MathMax CD-ROM	InterAct Math Online Exercises	Printed Test Bank/ Instructor's Resource Guide	
				Exercises	Chapter Review
Tape 4	4B	Sections 2.6 & 2.7		p. 622	p. 688